Table 1. Storage and drying effects on identification of rhizobia in soybean nodules by ELISA.

Storage temperature	Nondried	Air-dried	Freeze-dried	$LSD_{0.05}$
	ELISA units†			
	Strain 110			
Control‡	52 ± 15	--	--	
Ambient	62 ± 11	42 ± 9	44 ± 5	
5°C	--	31 ± 14	52 ± 11	
−5°C	49 ± 17	50 ± 12	54 ± 12	
−60°C	37 ± 13	--	46 ± 13	10
	Strain 1004			
Control	48 ± 12	--	--	
Ambient	64 ± 15	37 ± 17	39 ± 16	
5°C	--	35 ± 12	47 ± 16	
−5°C	65 ± 13	36 ± 17	53 ± 18	
−60°C	39 ± 17	--	29 ± 16	13
	Strain 587			
Control	35 ± 13	--	--	
Ambient	75 ± 14	47 ± 11	32 ± 10	
5°C	--	47 ± 10	35 ± 7	
−5°C	50 ± 7	53 ± 10	38 ± 8	
−60°C	37 ± 8	--	25 ± 3	8

† Absorbance values at 410 nm × 100.
‡ Controls were nodules which were serologically analyzed within 24 h after removal from the roots.

(LSD), and standard deviation as outlined by Steel and Torrie (1960).

Results and Discussion

Enzyme-linked immunosorbent assay (ELISA) is a colorimetric technique for identification of rhizobia. Positive ELISA reactions are characterized by intense color development, whereas, negative reactions develop color slowly. Negative reactions (resulting from the natural degradation of the enyzyme substrate) had readings of 10 ± 4 ELISA units. Data in Table 1 represent the quantitative measurement of color development associated with the various treatments.

Neither of the drying methods nor storage temperatures inhibited color development associated with positive reactions (Table 1). Color development for all treatment combinations was three- to six-fold greater than negative reactions. The nondried nodules stored at ambient temperature were generally more reactive than the other treatment combinations, however, this was probably a result of fungal contamination. In this study, nondried nodules stored at ambient temperature were more susceptible to fungal growth than the other treatment combinations. Means for the freeze-dried nodules stored at 5°C or −5°C were consistently similar to the mean of the control. However, since the other treatment combinations provided material suitable for ELISA, the similarity between freeze-dried nodules and the controls would not justify the expense of purchasing freeze-drying equipment. Additionally, air-dried and freeze-dried nodules lost pliability and required rehydration for maceration. Suspending nodules in saline solution for about 1 h restored pliability and provided material suitable for ELISA.

Conclusions

Identification of rhizobia in soybean nodules by ELISA was not inhibited by drying or by storage at the various temperatures for an extended period. For efficient resource management, we prefer freeze-dried nodules stored in sealed containers at −5°C. However, one or more of the alternative procedures may be more practical in some laboratories, depending on the number of samples collected and the time, personnel, and analytical facilities available to conduct serological analyses.

References

Hunt, P.G., A.G. Wollum, II, and T.A. Matheny. 1981. Effects of soil water on *Rhizobium japonicum* infection, nitrogen accumulation, and yield of Bragg soybean. Agron. J. 73:501–505.

Hunt, P.G., T.A. Matheny, A.G. Wollum, II, D.C. Reicosky, R.E. Sojka, and R.B. Campbell. 1983. Effect of irrigation and *Rhizobium japonicum* strain 110 upon yield and nitrogen accumulation and distribution of determinant soybeans. Commun. Soil Sci. Plant Anal. 14:223–238.

Lindemann, W.C., and C.W. Ham. 1979. Soybean plant growth, nodulation, and nitrogen fixation as affected by root temperature. Soil Sci. Soc. Am. Proc. 43:1134–1137.

Matheny, T.A., and P.G. Hunt. 1983. Effects of irrigation on accumulation of soil and symbiotically-fixed N by soybean grown on a Norfolk loamy sand. Agron. J. 75:719–722.

Skrdleta, V., and H. Mareckova. 1971. Serotyping of fresh and/or dried soybean nodules. Zentral-blatt fur Bakteriologie Parasitenkunde, Infektionskrankheiten und Hygiene Abs. II 7:656–658.

Somasegaran, P., R. Woolfenden, and J. Halliday. 1983. Suitability of oven-dried root nodules for *Rhizobium* strain identification by immunofluorescence and agglutination. J. Appl. Bacteriol. 55:253–261.

Speidel, K.L., and A.G. Wollum, II. 1980. Evaluation of leguminous inoculant quality: A manual. N.C. State Univ. Tech. Bull. 266.

Steel, R.G.D., and J.H. Torrie. 1960. Principles and procedures of statistics. McGraw-Hill Book Co., Inc., New York.

CALCULATIONS OF ERROR VARIANCES WITH STANDARDIZED VARIOGRAMS[1]

A. W. Warrick and D. E. Myers[2]

Abstract

Sample error calculations are presented for five common variogram models: exponential, spherical, Gaussian (double exponential), Langmuir (Michaelis-Menton) and linear. Numerical results for standardized variograms are presented in a 9 × 10 table useful for finding the maximum estimation (kriging) variance for a rectangular pattern valid for any spacing, slope, integral scale, etc., for zero nugget. Results are calculated based on the use of four or twenty-five nearest neighbors.

Additional Index Words: **geostatistics, kriging, variograms, sampling.**

Warrick, A.W., and D.E. Myers. 1987. Sample error variances with standardized variograms. Soil Sci. Soc. Am. J. 51:265–268.

Determination of the proper number of samples and the proper site locations is of widespread interest for a variety of sampling problems. Not only is economy of effort important, but also the variance of the error associated with whatever scheme is chosen. Burgess et al. (1981) and McBratney and

[1] Support was, in part, by Western Regional Research Project W-155. Technical Paper 4204, Arizona Agric. Exp. Stn., Tucson, AZ 85721. Received 25 April 1986.

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Table 1. Variogram models considered.

Model	Variogram $\gamma_\mu(H,B)$
1. Exponential	$1 - \exp(-H/B)$
2. Spherical	$1.5(H/B) - 0.5(H/B)^3$, $H > B$
	1 , $H > B$
3. Gaussian	$1 - \exp(-(H/B)^2]$
4. Langmuir (Michaelis-Menton)	$(H/B)/(1 + H/B)$
5. Linear	H/B

Webster (1983) considered optimal designs utilizing regionalized variables. They demonstrated that for an isotropic system, a square pattern had an estimation variance (kriging variance) comparable to that for a triangular pattern of the same sampling density. Their work clearly relates density of samples to reliability of estimates at an unmeasured site, more specifically the variance at the midpoint of the rectangle formed by four points on a sampling grid.

Our objective here is to calculate error variances based on standardized variograms. Numerical values will be given for estimation variances based on common models, generalized for nuggets, sills, integral scales, or correlation lengths. The results are presented in a tabular form which may be read directly to determine the error variance for most settings. Exponential, spherical, Gaussian (double exponential), Langmuir (Michaelis-Menton), and linear models are considered.

Theory

Table 1 lists common variogram models. The first four, the exponential, the spherical, Gaussian (double exponential), and Langmuir all have a sill of 1 and nugget of zero and are defined in terms of the ratio H/B, where B is related to the range or integral scale as appropriate. The "Langmuir" equation is of the same form as the Langmuir adsorption isotherm from surface chemistry as well as the Michaelis-Menton reaction rate from microbiology. The linear model has no sill. The first four models are appropriate for second order stationary processes.

Suppose H and B are measured in the grid of unit characteristic length to the right in Fig. 1, but we are interested in interpolations or predictions with the "real" system to the left with a characteristic length x_0. Without loss of generality assume the variogram for the real system is

$$\gamma(h,b) = C_0 + C_1\gamma_u(H,B), \qquad h > 0$$
$$= 0, \qquad h = 0 \qquad [1]$$

with h and b measured in the real system and C_0 and C_1 the nugget and slope. The proper definitions of H and B in terms of h and b are

$$H = h/x_0 \qquad [2]$$

$$B = b/x_0 \qquad [3]$$

with x_o a scaling length which corresponds to the unit length in the scaled system. We will consider the system to be isotropic for simplicity with calculations assumed to be nondirectional.

The equations for determining the weights for punctual kriging are (cf. Burgess and Webster, 1980)

$$\sum_j \lambda_j\gamma_{ij}(h,b) + \mu = \gamma_{io}(h,b),\ i = 1, \ldots N \qquad [4]$$

$$\sum_j \lambda_j = 1 \qquad [5]$$

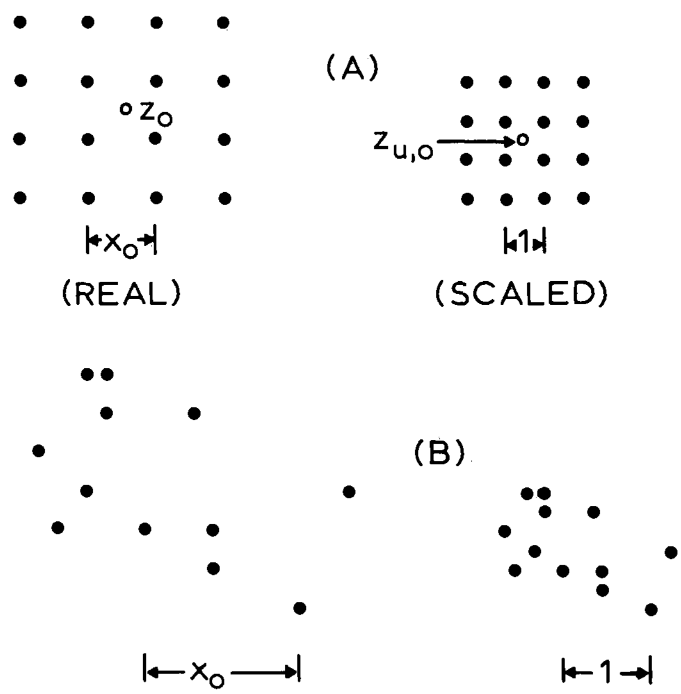

Fig. 1. Regular (A) and irregular (B) sets of location sites and scaled location sites.

where the sums are over the N points used for interpolation, λ_j are the weights and $\gamma_{ij} = \gamma(x_i - x_j)$. The μ is a Lagrange multiplier and γ_{io} is taken between point "i" and the estimation point.

Substitution of γ from Eq. [1] into Eq. [4] results in

$$-C_0\lambda_i + \Sigma\, \lambda_j[C_0 + C_1\gamma_{u,ij}(H,B)] + \mu$$
$$= C_0 + C_1\gamma_{u,io}(H,B)\,.$$

By Eq. 5, an equivalent form is

$$-C_0^*\lambda_i + (1 - C_0^*) \sum_j \lambda_j\, \gamma_{U,ij}(H,B)$$
$$+ \mu/(C_0 + C_1) = (1 - C_0^*)\, \gamma_{U,io} \qquad [6a]$$
$$i = 1 \ldots N$$

where C_0^* is a relative nugget defined as a fraction of the sill

$$C_0^* = C_0/(C_0 + C_1) \qquad [6b]$$

A comparison of Eq. [4] and [6] shows immediately the well known result that the weights λ_i are unaffected by a multiplicative factor C_1. For the first four model types of Table 1, the C_1 corresponds to the variance if no nugget is present. For the linear model, C_1 is the slope.

Also, the Lagrange multiplier (μ) follows by comparing Eq. [4] and [6]:

$$\mu = (C_0 + C_1)\, \mu_u\,. \qquad [7]$$

As the kriging variance is $\sigma_K^2 = \mu + \Sigma_j\, \lambda_j\gamma_{jo}$, then the relationship of σ_K^2 to the generalized $\sigma_{u,K}^2$ is

$$\sigma_K^2 = (C_0 + C_1)\sigma_{U,K}^2 \qquad [8]$$

where μ_u and $\sigma_{u,K}^2$ correspond to the unit system and $\sigma_{U,K}^2$ is a function of C_0^*.

Thus, if we have available λ_i and $\sigma_{u,K}^2$ for the unit system, those for the real system easily follow provided

$$H/B = h/b \qquad [9]$$

which is assured if H and B are scaled by x_0, i.e. Eq. [2] and [3] are satisfied.

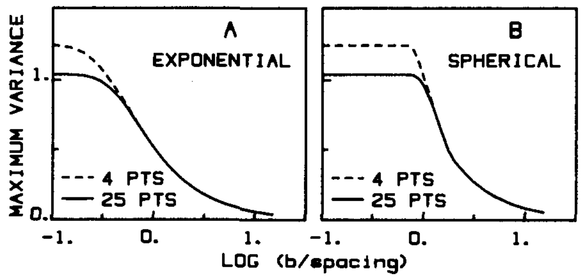

Fig. 2. Maximum standardized estimation variance for exponential and spherical variograms.

Numerical Results and Examples

Equation [8] can be used to find general values of kriging variance. For example, consider σ_K^2 and $\sigma_{K,u}^2$ corresponding to Z_o and $Z_{u,o}$ of the unsampled location in Fig. 1A. The center of the unsampled square within a regular grid has the largest kriging variance of the interior and is that considered by Burgess et al. (1981) and McBratney and Webster (1983). If $\sigma_{K,u}^2$ is known and Eq. [9] is satisfied, σ_K^2 is given by Eq. [8]. With this in mind, Fig. 2 was prepared, based on the 4 or the 25 nearest neighbors and $C_0 = 0$. Part A is for an exponential model distribution and Part B for a spherical. In each case the maximum relative variance ($\sigma_{K,u}^2$) is largest for small values of b/x_0 (spacing) and decreases as b/x_0 increases. The small b/x_0 corresponds to samples outside of the range of influence. For this case the best estimate reduces to the classical result $Z_i^* = \bar{Z} + \epsilon_i$ for which the variance estimate is

$$\mathrm{Var}(Z_i^*) = \sigma^2 (1 + 1/n) \qquad [10]$$

with n the number of samples used in the estimate and σ the population variance. Thus, the limiting relative variance is $1 + 1/n$ resulting in 1.25 and 1.04 for 4 and 25 samples, respectively. From a practical viewpoint the estimation variance need not be more than the population variance, because for independent samples, all samples can be used in Eq. [10] reducing $1 + 1/n$ to approximately 1.

If b/x_0 is large, the nearest neighbors dominate the estimate of Z_i and the relative variance goes to zero. A comparison of the results for 4 vs. 25 points reveals larger kriging variances for 4 points as expected for any particular b/x_0 although differences are insignificant for larger b/x_0.

Results for all the variograms considered are given in Table 2. The relative variance for Models 1 to 4 range from limiting values of $1 + 1/n$ at b/x_0 small to 0 for b/x_0 large. For the B (= b/x_0) values as defined in Table 1, the decrease for Langmuir form is very gradual, the exponential less so and the Gaussian very sharp.

Table 3 was prepared for a nonzero nugget and a spherical model. Maximum estimation variance is given as a function of b/x_0 using the nearest neighbors for a spherical model with values of $C_0^* = C_0/(C_0 + C_1)$ as 0, 0.05, 0.1, 0.25, 0.5, and 0.75.

As an example application, consider the estimation variance for thickness of cover loam at Plas Gogerdon after Burgess et al. (1981, esp. Fig. 3). The variogram is assumed isotropic and spherical

Table 2. Maximum estimation variance for a square grid spacing of x_0 for five variogram models. Estimation variance is presented for 4 and 25 nearest neighbors.

	Exponential		Spherical		Gaussian		Langmuir		Linear	
b/x_0	4	25	4	25	4	25	4	25	4	25
0.0	1.25	1.04	1.25	1.04	1.25	1.04	1.25	1.04	∞	∞
0.1	1.25	1.04	1.25	1.04	1.25	1.04	1.06	0.95	5.61	5.51
0.2	1.20	1.03	1.25	1.04	1.25	1.04	0.92	0.86	2.80	2.76
0.5	0.85	0.83	1.25	1.04	0.99	0.94	0.65	0.64	1.12	1.10
1.0	0.51	0.51	1.02	0.96	0.25	0.15	0.43	0.43	0.56	0.55
2.0	0.27	0.27	0.45	0.42	0.03	0.00	0.25	0.25	0.28	0.28
5.0	0.11	0.11	0.17	0.17	0.00	0.00	0.11	0.11	0.11	0.11
10.0	0.06	0.06	0.08	0.08	0.00	0.00	0.06	0.05	0.06	0.06
∞	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 3. Standardized kriging variances using 25 nearest neighbors and a spherical model.

	$C_0/(C_0 + C_1)$					
b/x_0	0	0.05	0.1	0.25	0.5	0.75
0	1.04	1.04	1.04	1.04	1.04	1.04
0.1	1.04	1.04	1.04	1.04	1.04	1.04
0.2	1.04	1.04	1.04	1.04	1.04	1.04
0.5	1.04	1.04	1.04	1.04	1.04	1.04
1.0	0.96	0.96	0.97	0.99	1.01	1.03
2.0	0.42	0.47	0.51	0.65	0.83	0.97
5.0	0.17	0.22	0.28	0.43	0.66	0.87
10.0	0.08	0.14	0.20	0.36	0.60	0.82
∞	0.00	0.05	0.10	0.25	0.50	0.75

$$\gamma(h) = 187 + 604\,[1.5(h/101) - 0.5(h/101)^3], \quad h < 101 \text{ m} \qquad [11]$$
$$= 791, \; h > 101 \text{ m}\,.$$

For a spacing of $x_0 = 25$ m, we have $b/x_0 = 101/25 = 4.04$. From Table 3 for $C_0/(C_0 + C_1) = 0.236$, we read $\sigma_{K,U}^2 \approx 0.43$. From Eq. [8], σ_K^2 is about

$$\sigma_K^2 \approx (187 + 604)(0.43) = 340$$

which agrees reasonably well with their value. Similarly for $x_0 = 50$ m, we find $\sigma_{u,K}^2 \approx 0.65$ and $\sigma_K^2 \approx 510$.

Values are also given in Table 2 for the linear model. Of course for the linear model, no sill exists and the variance is undefined. Thus, as b/x_0 becomes small, $\sigma_{K,u}^2$ increases without bounds. Again we can compare results with Burgess et al. (1981, esp. Fig. 1). For a spacing of 2 units and b = 1, $\sigma_{K,u}^2$ (for $b/x_0 = 0.5$) is about 1.1 which compares favorably.

Discussion

Numerical results have been presented in tabular form from which the estimation (kriging) variance may be read directly. Results are valid for all rectangular spacings, and as appropriate, for all integral scales, slopes, range, and sills for the five common variogram forms of Table 1. The scaling relationships are also given with respect to weights and for arbitrary sampling patterns. The result for Eq. [8] is valid for an irregular pattern as shown in Fig. 1B as well as for a regular grid.

Obviously, the reliability of the estimation variance results depends on the accuracy of the variogram. If the variogram is uncertain, the results will also be unreliable. However, the sensitivity of results as affected by changes in the variogram parameters follow from these results.

References

Burgess, T.M., and R. Webster. 1980. Optional interpolation and isarithmic mapping of soil properties I. The semi-variogram and punctual kriging. J. Soil Sci. 31:315–331.

Burgess, T.M., R. Webster, and A.B. McBratney. 1981. Optimal interpolation and isarithmic mapping of soil properties IV. Sampling strategy. J. Soil Sci. 32:643–659.

McBratney, A.B., and R. Webster. 1983. How many observations are needed for regional estimations of soil properties? Soil Sci. 135:177–183.

ANALYTICAL SOLUTION FOR PUNCTUAL KRIGING IN ONE DIMENSION[1]

PARICHEHR HEMYARI AND D. L. NOFZIGER[2]

Abstract

The solution to the system of equations required for punctual kriging in one dimension was determined analytically for a linear semivariogram. The solution indicates that kriging in this situation is identical to linear interpolation between the two closest neighbors. The kriged values are independent of the coefficients in the linear semivariogram model. The estimation variance is linearly dependent upon the coefficients in the model. Although no analytical solution was obtained for the spherical and exponential semivariance models, kriged values for these models were found to be essentially the linearly interpolated values. These results provide insight into the behavior of the kriging estimator. They can be used to save substantial numerical computation and to guide the user in selecting the neighborhood of points used in one-dimensional kriging.

Additional Index Words: **geostatistics, semivariogram, interpolation.**

Hemyari, P., and D.L. Nofziger. 1987. Analytical solution for punctual kriging in one dimension. Soil Sci. Soc. Am. J. 51:268–269.

IN RECENT YEARS, SOIL SCIENTISTS have used the kriging technique to estimate soil parameters. Kriging is based on the theory of regionalized variables developed in the 1960s by Matheron. Gambolati and Volpi (1979), Burgess and Webster (1980), and Vieira et al. (1983) discuss the technique and provide examples of its use. Kriging is a form of weighted averaging in which the weights are chosen such that the error associated with the estimate is less than for any other linear sum. The weights are determined by solving a system of linear equations. The weights depend upon the location of the points used in the estimation process and upon the structure of the variance of the parameter as reflected in the semivariogram. In general, the system of equations is solved numerically. Researchers are frequently interested in the influence of the number of points used in the estimation process and of uncertainty in the semivariogram model parameters upon the kriged values and associated estimation variance. These questions can be answered explicitly for one-dimensional kriging when the semivariogram model is linear. Brugess et al. (1981) provide part of these answers for a one-dimensional system with uniformly spaced measurements and a linear semivariogram with no nugget effect.

The objectives of this paper are to present an analytical solution to the system of equations for kriging in one dimension for a linear semivariance model, to determine the sensitivity of the kriged values and the estimation variances to the parameters in the linear semivariance model, and to compare the solution for the linear semivariance model with solutions for the spherical and exponential models.

Analytical Solution

The estimated value determined by kriging of a parameter Y at location x is given by

$$Y^*(x) = \Sigma_{i=1}^{N} w_i y(x_i) \quad [1]$$

where $y(x_i)$ for $i = 1,2,\ldots,N$ are measured values of the parameter and w_i is the weight for each measured value. The estimation variance $\sigma^2(x)$ is given by

$$\sigma^2(x) = \mu + \Sigma_{i=1}^{N} w_i \gamma(x_i, x) \quad [2]$$

where $\gamma(x_i, x)$ is the semivariance for values separated by the distance between points x and x_i and μ is the Lagrangian multiplier. Values of w_i and μ are obtained from the following system of linear equations

$$\Sigma_{j=1}^{N} w_j \gamma(x_i, x_j) + \mu = \gamma(x_i, x);\ i = 1,2,\ldots,N; \quad [3]$$

and

$$\Sigma_{j=1}^{N} w_j = 1\,. \quad [4]$$

Equations [3] and [4] define a system of $N + 1$ equations and $N + 1$ unknowns (w_i, $i = 1,2,\ldots,N$ and μ) which are solved simultaneously. For a linear semivariogram, $\gamma(h) = A + Bh$ where h is the distance between the points. Equation [3] then becomes

$$\Sigma_{j=1}^{N} w_j B|x_i - x_j| + \mu = B|x_i - x|\,. \quad [5]$$

If the values of x and x_i fall on a straight line the equations defined by Eq. [4] and [5] can be solved analytically. Solutions are given below for three cases. These solutions assume the data points are ordered such that x_i increases as i increases.

Case 1

$$\begin{aligned} &\text{If } x \le x_1 \text{ then } w_1 = 1\,; \\ &w_i = 0 \text{ for } i = 2,3,\ldots,N; \\ &\text{and } \mu = B(x_1 - x)\,. \end{aligned} \quad [6a]$$

Case 2

$$\begin{aligned} &\text{If } x_k < x \le x_{k+1} \text{ for } k \text{ in the range of 1 to } N \text{ then} \\ &w_k = (x_{k+1} - x)/(x_{k+1} - x_k)\,; \\ &w_{k+1} = (x - x_k)/(x_{k+1} - x_k)\,; \\ &w_i = 0 \text{ for } i < k \text{ or } i > k + 1\,; \\ &\text{and } \mu = 0\,. \end{aligned} \quad [6b]$$

Case 3

$$\begin{aligned} &\text{If } x > x_N \text{ then } w_N = 1 \\ &w_i = 0 \text{ for } i = 1,2,\ldots,N - 1; \\ &\text{and } \mu = B(x - x_N)\,. \end{aligned} \quad [6c]$$

These solutions can be verified by substituting them into

[1] Contribution from the Dep. of Agron. Oklahoma Agric. Exp. Stn. Journal Manuscript no. 4930. Received 23 Dec. 1985.

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